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ANALYSIS AND PROOF OF THE RIEMANN HYPOTHESIS

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Abstract: We propose a proof for the Riemann Hypothesis by dividing the Dirichlet eta function into a main term and a remainder term, focusing primarily on the behavior of the remainder in the critical strip *(*$0 < \sigma < 1$ *where* $s = \sigma + it$*)*. Then, we express the Riemann zeta function using the same decomposition and show that its main term cannot vanish at the nontrivial zeros. Finally, we focus on the limit on the main terms as $|\lim k \to \infty \ \ \zeta_k(s_0)/\zeta_k(1-s_0)|$.

Keywords and Phrases: Riemann hypothesis, proof, the Dirichlet eta function, the remainder term of the Dirichlet eta function.

2020 Mathematics Subject Classification: Primary 11M26.

1. Introduction

We hope you will enjoy reading this file which prepared very rigorously which is pretty fluent. First, let us introduce some basic properties of the Riemann zeta function *(let us call it only zeta function here)* and some well-known arts to make the arguments also accessible to a nonspecialist. The zeta function $\zeta(s)$ is defined as below with the complex variable $s = \sigma + it$ in the half-plane as $\sigma > 1$ where σ and t are real numbers. Here, it is an absolutely convergent series.

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots \quad \text{for } \sigma > 1 \tag{1.1}$$

With the analytic continuity, this function can become convergent in the whole complex plane.